Design of a Nature-like Fractal Celebrating Warp-knitting

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Abstract

In earlier work we created a new textile pattern which was derived from the well-known houndstooth pattern which originates from weaving with twill binding. The new pattern became interesting, both mathematically and aesthetically because it was a fractal. Now we are turning our attention to another basic fabric construction method: warp-knitting. We develop a recursive algorithm and explore the properties of the result. We also develop an attractive fashion item based on the new pattern, to be presented at Bridges.

Introduction

First we explain what warp-knitting is and in which sense we take inspiration from it. The typical characteristic of knitting is that the threads form loops, each loop being pulled through an existing loop. Roughly speaking, there are two main approaches to knitting, called weft-knitting and warp-knitting. The well-known hand-knitting is a special case of weft-knitting, for example a single yarn being knitted from left to right and then from right to left. In warp-knitting however, the yarn moves in the length-direction of the fabric in a zigzag manner [6]. So, unlike weft-knitting, a warp knitted fabric is composed of many yarns, not just one.

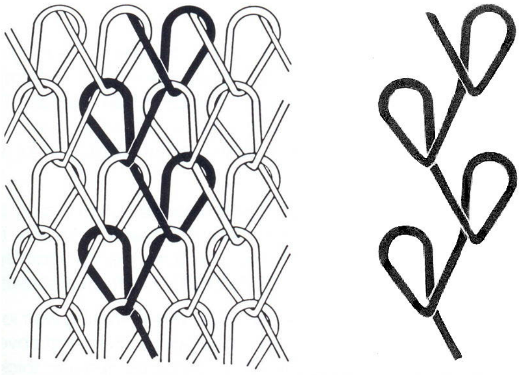

Figure 1: *Warp knitted fabric (left) and one thread thereof (right).*

Earlier work by Bernasconi, Bodie and Pagli on algorithmic knitting [1] demonstrates the power of recursion as a programming technique for knitted fractals (we use recursion as an essential tool too). The work of the present paper is the result of a new cooperation between TU/e, Jiangnan University and by-wire.net which was initiated during the DeSForM2013 conference in Wuxi. Whereas the Industrial Design Department in Eindhoven has strength in wearable senses and in generative design, the Engineering Research Center of Knitting Technology at Jiangnan University, Wuxi is specialised in warp-knitting. We share an interest in textile design and algorithmic pattern design, witnessed by results such as [4, 3, 2].

In Figure 1 (source of left figure: Wikimedia Commons) the basic principle of warp-knitting is given. One yarn is singled out and this one yarn with its loops is taken as the inspirational source for the fractal to be designed. But first we explain a bit of fractal theory.

How to make a fractal

We take inspiration from line fractals such as the Koch fractal and the dragon curve. Lindenmayer systems [5] are often used to describe the growth of fractal plants. This works with substition, e.g. a forward move `F` can be replaced by `F-F++F-F`. As a formal rule: `F` → `F-F++F-F`. The idea is to apply the rule repeatedly (to all `F` simultaneously). Starting from `F` we get `F-F++F-F`, then `F-F++F-F-F-F++F-F++F-F++F-F-F-F++F-F`, and so on. Interpreting the symbols as turtle graphics commands, one gives `F` the meaning of drawing forward, `+` to turn right 60°, and `-` to turn left 60° and then this Lindenmayer system describes the Koch fractal.

The warp-knitting fractal

We show the approximations of our new fractal for nesting levels N = 0,1,2,3 and 4 in Figure 2. These have been created using a recursive algorithm and a turtle-graphic system, in a similar way in which one makes the Koch fractal. The lines in Figure 2 are drawn starting at the bottom of the figure with the turtle pointing upward. For the second line of Figure 2, the turtle made two loop pairs. In this way we get loops similar to the single yarn of Figure 1 (the loops are not nicely rounded yet, but we will repair that later).

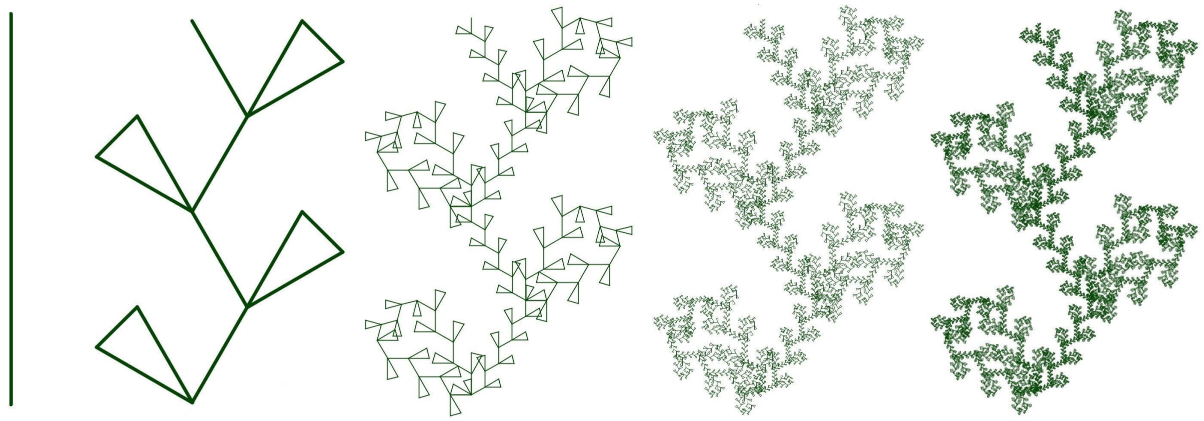

Figure 2: Approximations of the warp-knitting fractal for $N = 0, 1, 2, 3$ and 4.

This gives us a recipe for a fractal: draw a looped line, but whenever the basic recipe tells us to move forward, we move forward while doing a few loop pairs. More precisely: we do 3 loop pairs for the first "forward", 2 for the next (it is shorter by a factor of $2 \sin 15°$), then 3 again, and 4 for the last "forward". And then we repeat in a glide-mirrored fashion. The numbers are chosen after experimentation: 2 for the shortest line, 3 inside the loops (where the corners would become messy otherwise) and 4 for the last move. The recipe is related to the Lindenmayer rule $\mathtt{F} \to \mathtt{-F^3\text{-}F^2\text{-}F^3\text{-}F^4F^3\text{+}F^2\text{+}F^3\text{+}F^4\text{+}}$ where $\mathtt{F}^2$ abbreviates `FF`, $\mathtt{F}^3$ abbreviates `FFF` and so on and where the four minus signs represent left turns of 30°,105°, 105° and 90° respectively; the plus signs represent right turns of 105°,105°, 90° and 30° (to specify the exact lengths we would need the more powerful formalism of parametric L-systems). In practice we use the Oogway library in Processing [2]. This also allows us to fine-tune the scaling factors of subfigures and explore aesthetic effects.

Fractal dimension

Replacing a line of length 1 by a loop pair, it turns into six segments of length $\frac{1}{3}\sqrt{3} \approx 0.577$ and two of length $\frac{2}{3}\sqrt{3}\sin 15° \approx 0.299$. If we replace it by three loop pairs, it turns into 18 segments of length $\approx 0.577/3 \approx 0.19$ and 6 of length $\approx 0.299/3 \approx 0.10$. So one line is replaced by 24 segments of a (weighted) average of length 0.17. In the fractal, most lines are replaced by three loop pairs, but there are also those which are replaced by two loop pairs or by four. To estimate the dimension we pretend each line is replaced by three double loops, so it is broken up in 24 segments of length ≈ 0.17. Writing n for the number of line segments, s for the scaling factor, $n = 24$ and $\frac{1}{s} = 1/0.17 = 5.9$ so $D \approx (\log 24)/(\log 5.9) = 1.8$. The fractal is almost two-dimensional, which is what we see in the rightmost line of Figure 2: the line almost appears to fill certain areas. This gives the line its natural appearance, like a plant. If we insist on avoiding approximations, we solve $n_1 \times s_1^D + n_2 \times s_2^D + n_3 \times s_3^D = 1$ where $n_1 = 12$, $s_1 = (\frac{1}{3}\sqrt{3})/3$, $n_2 = 8$, $s_2 = (\frac{1}{3}\sqrt{3})/4$, $n_3 = 4$, and $s_3 = (\frac{2}{3}\sqrt{3}\sin 15^\circ)/2$. Using Mathematica's FindRoot we get $D = 1.79659$.

Back to fashion

We promised to make rounded loops, which we achieve using `beginSpline` and `endSpline` in Oogway [2]. This strengthens the nature-like appearance and even for low N it resembles a vine plant now (Figure 3, left). The next step is designing a real fashion item: an elegant lady's dress. We used a combination of knitting (the jersey substrate) and textile printing (the fractal line); special thanks go to Pauline Klein Paste of HKU (Utrecht School of Arts). The pattern can be seen in Figure 3 (center) and the dress in Figure 3 (right) and Figure 4. An interesting question is whether the new pattern can be really machine-knitted. It will also be interesting to see what happens if we involve multiple threads (we leave these questions as options for future research). We shall bring the dress to Bridges Seoul.

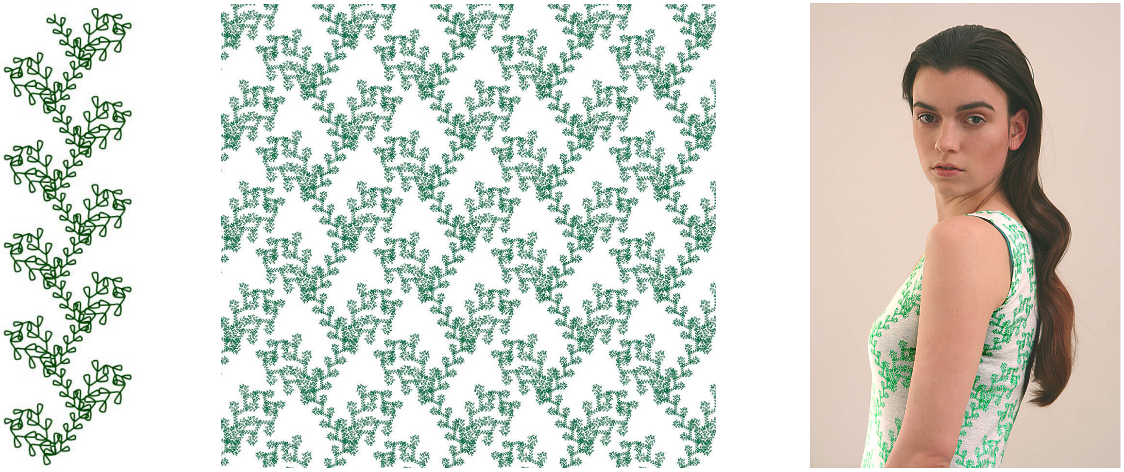

Figure 3: *Spline-based line (left), pattern (center) and lady's dress with pattern of line fractal(right), (Model Charlotte Geeraerts, Make-up artist Lana Houthuijzen, Photographer Katinka Feijs).*

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Figure 4: *Lady's dress with green fractal (Model Charlotte Geeraerts, Make-up artist Lana Houthuijzen, Photographer Katinka Feijs).*

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Preface

Welcome to Bridges Seoul 2014, the 17th annual Bridges conference! This year we are thrilled to build our longest bridge yet, making our way to Asia for the first time after our many travels across North America and Europe. We look forward to the opportunity to interact with a new community of participants. We hope also to foster excitement about art and mathematics by engaging with the Korean public in cooperation with our hosts, the Gwacheon National Science Museum.

The Bridges board of directors is grateful to Professor Ingrid Daubechies, President of the International Mathematical Union, for suggesting that we hold Bridges in Seoul as a satellite conference to the 2014 International Congress of Mathematicians (Seoul ICM 2014), and for initiating the contact between Bridges, the ICM, and the Gwacheon National Science Museum. Based on several site visits and a series of meetings with the Gwacheon Museum Board of Directors and ICM organizers, we have assembled an international committee that has worked hard to make this conference a reality.

Gwacheon National Science Museum, which opened its doors in 2008, is the largest science museum in Asia, and one of the largest in the world. Through its highly interactive exhibits and permanent collections, it strives to communicate scientific knowledge to a broad audience in an accessible way. Bridges is naturally aligned with this point of view—from the beginning, the conference has explored how to use art to talk about mathematics and aid the acquisition and retention of mathematical ideas. Conversely, we are pleased to observe that the museum's vision statement demands that exhibitions be founded upon STEAM (Science, Technology, Engineering, Art, Mathematics), and not just STEM.

The Bridges Organization's educational goals are clearer than ever this year, as we simultaneously inaugurate MoSAIC (Mathematics of Science, Art, Industry, Culture), a series of mathematical art mini-conferences sponsored by the Mathematical Sciences Research Institute (MSRI). MSRI is one of the world's preeminent organizations for collaborative research. MoSAIC events can be organized anywhere, and funding is available for guest speakers and hands-on workshops. More information on MoSAIC can be found at `www.mosaicmathart.org`.

Mathematics, art, and science all date far back to the prehistory of mankind and have in common that they involve abstractions from observations of nature. Various types of patterns and structures naturally arise in these three fields. It is a central theme of Bridges conference papers to elucidate and depict such structures, so this year's setting in the Science Museum gives us a particularly appropriate environment in which to ponder the roots of the structures found in math, art, and science.

This year's Program Committee Chair is Gary Greenfield, with George Hart and Reza Sarhangi serving as co-editors of the proceedings. Under Gary's direction, a committee comprising more than forty experts from around the world provided extensive and rigorous reviews to submissions in three categories—regular papers, short papers, and workshop papers—and offered further feedback and advice to authors of accepted papers in order to improve their final versions. This process ultimately yielded the 37 regular papers, 44 short papers, and 8 workshop papers that are included in this volume. The editors would like to acknowledge the dedicated reviewing efforts of Mara Alagic, Bob Bosch, Paul Gailiunas, Craig S. Kaplan, Douglas M. McKenna, and Carlo Séquin who generously provided extra support. We thank all the authors, program committee members and other volunteers for their careful work.

An exhibition of mathematical art has been an annual feature of Bridges since 2001, and well over 100 artists contributed to this year's art exhibition. The list of contributors includes several newcomers from Japan, South Korea, and China, as well as artists from North and South America, Europe, Africa, and Australia. A wide variety of artistic media are represented in the exhibition, including 2D and 3D digital prints, painting, beadwork, ceramics, wood, metal, quilting, and paper folding. Artists drew inspiration from the mathematics of fractals, polyhedra, non-Euclidean and four-dimensional geometry, tiling, knot theory, magic squares, and more. This year Katie McCallum and Robert Fathauer served as co-curators of the exhibition, and were joined by Anne Burns, Nat Friedman and Chaesoon Kwon to make up the jury. The print catalog was edited by Conan Chadbourne, Robert Fathauer, and Katie McCallum.

Once again, we are pleased to welcome you to this year's conference. We hope that you find insight and inspiration in the papers in this volume, in the diverse events that make up Bridges, and in our wonderful location.

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